MATH 1920, Fall 2017
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Sections: 201, 202

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Section 18.2 Review

Section 18.2 Summary

- The **boundary** of a surface S is denoted ∂S.
- The **boundary orientation** of ∂S is defined as follows: If you walk along the boundary in the positive direction with your head pointing in the normal direction, then the surface is on your left.
- State **Stokes' Theorem**:

- The definition of vector potential is: ____________________________
- Surface independence: If $\boldsymbol{F} = \text{curl}(\boldsymbol{A})$, then the flux of $\boldsymbol{F}$ through a surface S depends only on the oriented boundary ∂S and not on the surface itself. Verify this using Stokes' theorem:

- If S is closed and $\boldsymbol{F} = \text{curl}(\boldsymbol{A})$, then

$$\int\int_S \boldsymbol{F} \cdot d\boldsymbol{S} =$$

Section 18.2 Additional Exercises

1. Verify Stokes' Theorem for the vector field $\boldsymbol{F} = \langle y, x, x^2+y^2 \rangle$ where S is the upper hemisphere $x^2+y^2+z^2 = 1, z \geq 0$ with an upward-pointing normal.

2. Calculate curl($\boldsymbol{F}$) and then apply Stokes' Theorem to compute the flux of curl($\boldsymbol{F}$) through the given surface using a line integral. For $\boldsymbol{F} = \langle yz, -xz, z^3 \rangle$, that part of the cone $z = \sqrt{x^2 + y^2}$ that lies between the two planes $z = 1$ and $z = 3$ with upward-pointing unit normal vector.

3. Apply Stokes' theorem to evaluate $\oint_C \boldsymbol{F} \cdot d\boldsymbol{r}$ by finding the flux of curl($\boldsymbol{F}$) across an appropriate surface. For $\boldsymbol{F} = \langle y, -2z, 4x \rangle$, C is the boundary of that portion of the plane $x + 2y + 3z = 1$ that is in the first octant of space, oriented counter clockwise as viewed from above.

4. Let S be the portion of the plane $z = x$ contained in the half-cylinder of radius R depicted in Figure 17. Use Stokes' theorem to calculate the circulation of $\boldsymbol{F} = \langle z, x, y + 2z \rangle$ around the boundary of S (a half-ellipse) in the counterclockwise direction when viewed from above. Hint: Show that curl($\boldsymbol{F}$) is orthogonal to the normal vector to the plane.

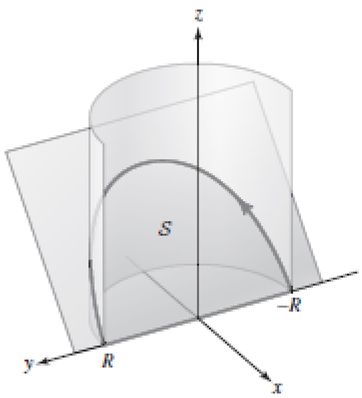

FIGURE 17

5. Let $\boldsymbol{F} = \langle y^2, x^2, z^2 \rangle$. Show that

$$\int_{C_1} \boldsymbol{F} \cdot d\boldsymbol{r} = \int_{C_2} \boldsymbol{F} \cdot d\boldsymbol{r}$$

for any two closed curves lying on a cylinder whose central axis is the $z-$ axis.

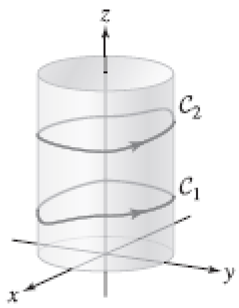

FIGURE 21

6. You know two things about a vector field $\boldsymbol{F}$:

(i) $\boldsymbol{F}$ has a vector potential $\boldsymbol{A}$ (but $\boldsymbol{A}$ is unknown).
(ii) The circulation of $\boldsymbol{A}$ around the unit circle (oriented counter-clockwise) is 25.
Determine the flux of $\boldsymbol{F}$ through the surface S in Figure 22, oriented with an upward-pointing normal.

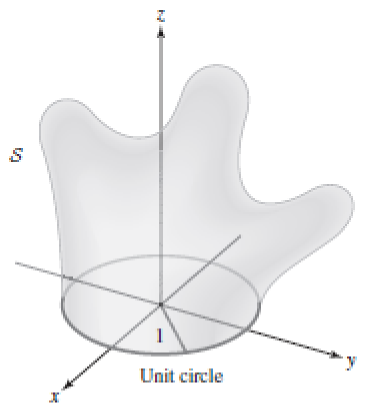

FIGURE 22 Surface S whose boundary is the unit circle.

7. Assume that f and g have continuous partial derivatives of order 2. Prove that

$$\oint_{\partial S} f\nabla(g) \cdot d\boldsymbol{r} = \int\int_S \nabla(f) \times \nabla(g) \cdot d\boldsymbol{S}.$$

8. Explain **carefully** why Green's theorem is a special case of Stokes' theorem.